412TW-PA-15240

Logistic Regression Model on Antenna Control Unit Autotracking Mode

DANIEL T. LAIRD

AIR FORCE TEST CENTER
EDWARDS AFB, CA

OCTOBER 20 2015

4
1
2
T
W

Approved for public release; distribution is unlimited.
412TW-PA-15240

AIR FORCE TEST CENTER
EDWARDS AIR FORCE BASE, CALIFORNIA
AIR FORCE MATERIEL COMMAND
UNITED STATES AIR FORCE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
20 OCT 2015

2. REPORT TYPE
Technical Brief

3. DATES COVERED *(From - To)*
20 OCT 15 – 23 OCT 15

4. TITLE AND SUBTITLE
Logistic Regression Model on Antenna Control Unit Autotracking Mode

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Daniel T. Laird

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) AND ADDRESS(ES)
412th TW/ENI, Edwards AFB CA 93524

8. PERFORMING ORGANIZATION REPORT NUMBER
412TW-PA-15240

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)
N/A

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release A: distribution is unlimited.

13. SUPPLEMENTARY NOTES
CA: Air Force Test Center Edwards AFB CA CC: 012100

14. ABSTRACT
Over the past several years DoD has imposed constraints on test deliverables, requiring objective measures of test objective results, i.e., statistically defensible test and evaluation (SDT&E) methods and results. These constraints force tester to employ statistical hypotheses, analyses and modeling to assess test results objectively, i.e., on statistical metrics, analytical methods, probability of confidence complemented by, rather than relying solely on expertise, which is too subjective an approach. In this paper we discuss methods of objectifying testing via statistical modeling and decide for a null- or alternative-hypothesis. This paper will present an Antenna Auto-tracking model using Logistic Regression modeling. This paper presents an example of tools useful for employing a SDT&E methodology.

15. SUBJECT TERMS
Statistically Defensible Test & Evaluation (SDT&E), Statistical Modeling, Logistic Regression, Null-Hypothesis, Alternative-Hypothesis, Antenna Testing.

16. SECURITY CLASSIFICATION OF: Unclassified			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			412 TENG/EN (Tech Pubs)
Unclassified	Unclassified	Unclassified	None	13	19b. TELEPHONE NUMBER *(include area code)* 661-277-8615

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39.18

LOGISTIC REGRESSION MODEL ON ANTENNA CONTROL UNIT AUTOTRACKING MODE

By

DANIEL T. LAIRD

ABSTRACT

Over the past several years DoD imposed constraints on test deliverables, requiring objective measures of test results, i.e., statistically defensible test and evaluation (SDT&E) methods and results. These constraints force testers to employ statistical hypotheses, analyses and modeling to assess test results objectively, i.e., based on statistical metrics, analytical methods, probability of confidence complemented by, rather than solely on expertise, which is too subjective. In this and companion papers we discuss methods of objectifying testing. We employ an earth coordinate model and statistical modeling of telemetry (TM) tracking antenna[a] employing time-space position information (TSPI) and derived statistical measures for *tracking-error* and *auto-tracking mode*. Test data were statistically analyzed via analysis of covariance (ANCOVA) which revealed that the antenna control unit (ACU) under test (AUT) does not track statistically identically, nor as practically or efficiently in C-band while receiving data carriers in both S- and C-bands. The conclusions of this paper add support to that hypothesis. In this third of three papers[b] we use data from a range test, but make no reference to the systems under test as the purpose of this paper is to present an example of tools useful for employing a SDT&E methodology.

KEY TERMS

H_0, tracking mode, tracking error, TM, AGC, ACU, AT, Scan, Slew, GPS, TSPI samples, observables and predictors, ANOVA, ANCOVA, *F*-test, *t*-test, PDF, statistical model, inner-product, linear predictor, link function, logit, logistic regression, stochastic filter.

INTRODUCTION

Using statistical analyses we decided[c] to reject our test's null-hypothesis, H_0 on objective criteria derived from ANOVA and t-test. The test null hypothesis is:

H_0: *AUT tracking modes on C-band are statistically independent of data carrier*.

The AUT tracks on signal strength sampled via automatic gain control (AGC) circuits, and on controllable conic-scan (*Scan*) and antenna slew (*Slew*) rates. The test includes a C-band AUT tracking of C- and S-band carriers. Our measure of tracking is the *probability of autotrack mode* (AT) sampled from the AUT and regressed against AUT observed AGC and rate controls. Ideally the tracking algorithm should have no dependence on observed angles, and the tracking

[a] Rf. 3

[b] Rf. 1 & 2.

[c] Rf. 1, 2, 4, 7 & 8.

algorithm is assumed to control the antenna center pointing angles independent of angle to target, i.e., the conic-scan centering is controlled via scanned signal strength. Our work bears this out: the better models exclude angle data. This paper is an extension of previous analyses of tracking as a synthesis of a tracking model that predicts autotracking based on *tracking modes*. A full predictor *stochastic filter* can be built on the predictor updates and angle errors of a linear model, where error is based on difference of observed and calculated angles of a *tracking-error* model.[d] This paper presents an *auto-tracking*, or *mode* model.

LOGISTIC REGRESSION MODEL

TM signal power is modeled as bounded continuous parameters received and sampled from automatic gain control (AGC) circuits of the AUT. As well, azimuth and elevation angles are continuous parameters. The AUT employs two categorical *tangential*[e] or *rate* control parameters we denote *Scan* and *Slew*. Available data includes these parameters and others from the tracker ACU, as well as target TSPI, i.e., GPS and INS. Since a tracking mode should not depend on its local angle we do not include angles in the model. This was not merely a choice, but the result of model reduction from the full model. We tried various models before settling on this 'best model' using a linear predictor based on ACU parameters independent of target. Auto-track holds if both azimuth and elevation modes auto-track, all other modes are considered *not-auto-track*. Thus we have two response states, which we map into the response set, y:{0,1}. This two-state response is the reason for a *binary response*, which is transformed to a continuous response p:[0,1] and regressed against a *linear predictor* for our model. The tracking-error model has continuous response, *expected error* that is linearly regressed against angle and signal strength parameters.[f]

LOGIT of PROBABILITY:

The auto-tracking model is built on a *linear predictor* formed as an inner-product of an observable-control space and its regressed vector. The predictor form is

$$\boldsymbol{\beta}\bullet\mathbf{x} = \beta_0 + \Sigma\, \beta_k x_k \equiv bx. \qquad (1)$$

This resolves to scalar. $\boldsymbol{\beta}$ designates regressed coefficients and $\mathbf{x}$ are sampled vectors from the observable-control space; β_0 designates the predictor intercept. The term, bx is a standard statistical representation of this *inner-product*. In the literature bx is also designated as η. The *link function* is defined to yield a linear response against the predictor (which we verify when checking the model). Our link is the natural logarithm of *odds*:

$$\mathrm{link}(y) \equiv \ln(\mathrm{O}(p)) = \ln(p/1 - p);\ p \in [0,1]. \qquad (2)$$

This transforms the binary set y:{0,1} to a smoothed step function, or 'S-curve' bounding response to predictor to the interval [0,1]. Thus probability p (or of expectation μ) of the link of y is the logarithm of odds, or *log-odds*, also known as the *logit* of probability. Our model derives the *logit* of probabilities as the *linear predictor* of regression. We represent this:

---

[d] Ibid.
[e] Rf. 1, 7 & 8.
[f] 4 & 7.

$$y \leftrightarrow \text{logit}(p) \sim lm(bx). \qquad (3)$$

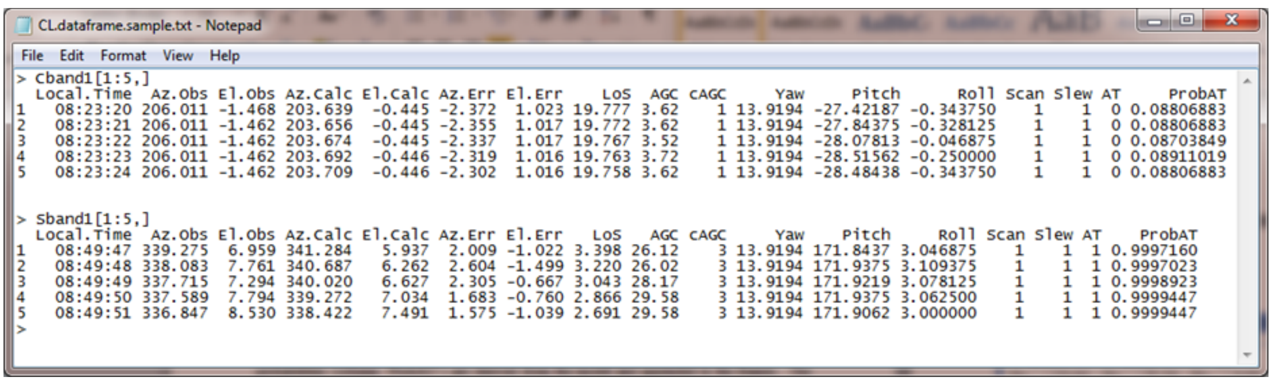
The *odds* is easily recovered via exponentiation:

$$\text{O}(p) = p/(1 - p) = e^{bx}. \qquad (4)$$

From this we derive a probability of odds, which is our probability of response to observation:

$$p(\text{O}) = (1 + e^{-bx})^{-1} = p(f(y)). \qquad (5)$$

For tracking observation, y estimates the autotrack mode parameter AT $\in \{1\}^+$, which is regressed against other ACU parameters to transform a binary state to continuous state space y:{0,1} $\rightarrow$ p:[0,1]. Regression parameters are collated in an R *dataframe*, shown in figure 1. We separate data by band. The probabilities (column ‘ProbAT’) are predicted by the model and appended to the frames post analysis. Each frame contains time from ACU and GPS-INS samples to align tracker and target data. We correlate at 1Hz. The observed angles and AGC values come from the ACU, the calculated angles and LoS are derived via geometric model[g] that translates TSPI global position, the attitude data is from the INS, the control data and AT, which is derived from the AUT Az/El track modes.


CL.dataframe.sample.txt - Notepad

File Edit Format View Help

```
> Cband1[1:5,]
  Local.Time  Az.Obs El.Obs Az.Calc El.Calc Az.Err El.Err    LoS  AGC CAGC     Yaw    Pitch      Roll Scan Slew AT     ProbAT
1   08:23:20 206.011 -1.468 203.639  -0.445 -2.372  1.023 19.777 3.62    1 13.9194 -27.42187 -0.343750    1    1  0 0.08806883
2   08:23:21 206.011 -1.462 203.656  -0.445 -2.355  1.017 19.772 3.62    1 13.9194 -27.84375 -0.328125    1    1  0 0.08806883
3   08:23:22 206.011 -1.462 203.674  -0.445 -2.337  1.017 19.767 3.52    1 13.9194 -28.07813 -0.046875    1    1  0 0.08703849
4   08:23:23 206.011 -1.462 203.692  -0.446 -2.319  1.016 19.763 3.72    1 13.9194 -28.51562 -0.250000    1    1  0 0.08911019
5   08:23:24 206.011 -1.462 203.709  -0.446 -2.302  1.016 19.758 3.62    1 13.9194 -28.48438 -0.343750    1    1  0 0.08806883


> Sband1[1:5,]
  Local.Time  Az.Obs El.Obs Az.Calc El.Calc Az.Err El.Err   LoS   AGC CAGC     Yaw    Pitch     Roll Scan Slew AT    ProbAT
1   08:49:47 339.275  6.959 341.284   5.937  2.009 -1.022 3.398 26.12    3 13.9194 171.8437 3.046875    1    1  1 0.9997160
2   08:49:48 338.083  7.761 340.687   6.262  2.604 -1.499 3.220 26.02    3 13.9194 171.9375 3.109375    1    1  1 0.9997023
3   08:49:49 337.715  7.294 340.020   6.627  2.305 -0.667 3.043 28.17    3 13.9194 171.9219 3.078125    1    1  1 0.9998923
4   08:49:50 337.589  7.794 339.272   7.034  1.683 -0.760 2.866 29.58    3 13.9194 171.9375 3.062500    1    1  1 0.9999447
5   08:49:51 336.847  8.530 338.422   7.491  1.575 -1.039 2.691 29.58    3 13.9194 171.9062 3.000000    1    1  1 0.9999447
>
```

Figure 1 Regression Parameters

For each band we have the two categorical *tangential* control parameters[h]: *conical scan rate* encoded in two levels and planar *slew rate* encoded in four levels: *Scan* $\in \{2\}^+$; *Slew* $\in \{4\}^{+}$[i]. In the models categorical parameter levels are *typed* as *factors*. The sampled data also contains continuous angle and AGC parameters. All angles are bounded modulo 360°: {Az.Obs, El.Obs, Yaw, Pitch, Roll} $\subset$ [0, 360], and AGC, which is a derived parameter from truncated minimum and maximum power, ranging from $0 \le \text{dB} \le 52$, of two tracking receivers for both C- and S-band signals. The *full general linear model (glm)*, with link from the binomial family, is represented in R as:

$$glm(\text{AT} \sim \text{AGC} + Slew + Scan, \text{family = binomial}). \qquad (6)$$

Time alignment correlates ACU and TSPI samples at 1Hz. Observed angles and AGC levels come from the ACU, the expected angles and line-of-sight (LoS) are derived via a

[g] Rf. 1.

[h] Rf. 7 & 8.

[i] $\{n\}^+ \equiv \{1, \ldots, n\}$; $\{n\} \equiv \{0, \ldots, n-1\}$.

geometric model[j] that translates TSPI global position to local azimuth and elevation angles of LoS between tracker and target. For reasons stated earlier, we do create our full model from all parameters, and employ different parameters creating the linear error model discussed in the companion paper on *tracking-error*. Figure 2a shows a statistical summary of the data, sans time and angles. We include angle errors for perspective. You can see that the mean AT count is consistent with probability (ProbAT) of the model, for both bands. It is also apparent why we do not use Yaw in modeling the target signal pattern – it is constant.

summary S&C-bands.txt - Notepad

File Edit Format View Help

```
> summary(Sband1[,6:17])
     Az.Err             El.Err            LoS              AGC              cAGC            Yaw       
 Min.   :-175.611   Min.   :-87.714   Min.   : 0.456   Min.   : 0.010   Min.   :1.000   Min.   :13.92  
 1st Qu.: -23.817   1st Qu.: -5.460   1st Qu.: 6.654   1st Qu.: 1.840   1st Qu.:1.000   1st Qu.:13.92  
 Median :   0.828   Median : -2.736   Median : 8.703   Median : 4.430   Median :1.000   Median :13.92  
 Mean   :  -9.808   Mean   : -4.605   Mean   : 9.943   Mean   : 8.125   Mean   :1.459   Mean   :13.92  
 3rd Qu.:   2.817   3rd Qu.: -1.052   3rd Qu.:14.188   3rd Qu.: 7.157   3rd Qu.:1.000   3rd Qu.:13.92  
 Max.   : 348.351   Max.   :  8.233   Max.   :21.255   Max.   :45.330   Max.   :5.000   Max.   :13.92  
     Pitch              Roll             Scan            Slew             AT             ProbAT        
 Min.   :-179.84   Min.   :-9.156   Min.   :1.000   Min.   :1.000   Min.   :0.000   Min.   :0.0002156  
 1st Qu.: -15.11   1st Qu.: 2.484   1st Qu.:1.000   1st Qu.:2.000   1st Qu.:0.000   1st Qu.:0.0138504  
 Median :  11.85   Median : 2.875   Median :1.000   Median :3.000   Median :0.000   Median :0.0579423  
 Mean   :  55.81   Mean   : 2.789   Mean   :1.225   Mean   :2.837   Mean   :0.273   Mean   :0.2729529  
 3rd Qu.: 171.69   3rd Qu.: 3.250   3rd Qu.:1.000   3rd Qu.:4.000   3rd Qu.:1.000   3rd Qu.:0.3831964  
 Max.   : 179.59   Max.   :10.438   Max.   :2.000   Max.   :4.000   Max.   :1.000   Max.   :1.0000000  

> summary(Cband1[,6:17])
     Az.Err             El.Err            LoS              AGC              cAGC          Yaw       
 Min.   :-355.331   Min.   :-79.188   Min.   : 0.622   Min.   : 1.160   Min.   :1.00   Min.   :13.92  
 1st Qu.: -18.751   1st Qu.: -4.250   1st Qu.: 6.601   1st Qu.: 3.930   1st Qu.:1.00   1st Qu.:13.92  
 Median :   0.241   Median : -1.581   Median :10.785   Median : 7.210   Median :1.00   Median :13.92  
 Mean   :  -5.007   Mean   : -2.874   Mean   :11.126   Mean   : 9.648   Mean   :1.48   Mean   :13.92  
 3rd Qu.:   4.306   3rd Qu.:  0.376   3rd Qu.:15.745   3rd Qu.:13.360   3rd Qu.:2.00   3rd Qu.:13.92  
 Max.   : 349.406   Max.   : 22.449   Max.   :23.996   Max.   :51.660   Max.   :5.00   Max.   :13.92  
     Pitch               Roll              Scan            Slew             AT              ProbAT        
 Min.   :-179.781   Min.   :-19.562   Min.   :1.000   Min.   :1.000   Min.   :0.0000   Min.   :0.0002933  
 1st Qu.: -20.641   1st Qu.:  2.125   1st Qu.:1.000   1st Qu.:1.000   1st Qu.:0.0000   1st Qu.:0.0280136  
 Median :   3.844   Median :  3.094   Median :1.000   Median :1.000   Median :0.0000   Median :0.0534819  
 Mean   :  40.032   Mean   :  3.071   Mean   :1.257   Mean   :1.689   Mean   :0.1549   Mean   :0.1548841  
 3rd Qu.: 169.141   3rd Qu.:  4.656   3rd Qu.:2.000   3rd Qu.:3.000   3rd Qu.:0.0000   3rd Qu.:0.1958546  
 Max.   : 180.000   Max.   : 15.328   Max.   :2.000   Max.   :3.000   Max.   :1.0000   Max.   :0.9980557  
```

Figure 2a Statistical Summary of Data

A simple count of the auto-tracks (sum 'AT') over the categorical parameters yields the table shown in figure 2b. Of the 5021 samples culled from the source data we have 968 auto-tracks (AT = 1) and 4053 non-auto-tracks (AT = 0). A quick estimate of probability of auto-track is 968/5021 ≈ .20.

[j] Rf. 1.

auto-tracks_C-&S-band.t...

File Edit Format View Help

Scan	Slew	Band	AT: 0	1
1	1	C	1484	455
		S	190	127
	2	C	305	1
		S	170	134
	3	C	243	44
		S	234	82
	4	C	0	0
		S	271	41
2	1	C	0	0
		S	0	0
	2	C	272	14
		S	0	0
	3	C	577	14
		S	0	0
	4	C	0	0
		S	307	56
Total			4053	968

Figure 2b Auto-track Counts Summary

This quick look indicates about a 20% probability of being in autotrack, thus *odds* of ¼, over all scan and slew rate combinations. Figure 2a shows the average autotrack over both bands is $(0.2730 + 0.1550)/2 \approx 0.2140$. Thus predicted probability is consistent with the AT counts over both and across band samples. From this table we can calculate the probability over bands, scan and slew. We see in figure 2b that at *Scan* level 1, *Slew* level 4 there are no tracks. So we don't need this level in our model. Also at *Scan* level 2 only *Slew* levels 2 and 3 auto-track and only over C-band carrier. Rather than eliminate these, we allow the regression and ANOVA[k] table to reveal their insignificance. To decide on 'the best' model we build our model and reduce it. We reduce by eliminating or recategorizing parameters with a high *p*-value, or a low probability of information. Note that the *p*-value is not the probability of track, but a probability of significance derived from a *t*-test (displayed as t or z value), that the modeled parameter coefficient contributes informative rather than random model results. Once we have our reduced model, we test it against statistical metrics to determine our model's 'goodness of fit'.

[k] Rf. 1.

C&S-band_LogRM.models.1.txt - Notepad

File Edit Format View Help

```
#Log-RM Log-odds/odds-ratio/probability Test Results:

#S-band:
> summary(Slogit)

Call:
glm(formula = AT ~ AGC + Scan + Slew, family = binomial, data = Sband1)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-3.9063  -0.3310  -0.2065   0.0012   2.8847

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -4.19102    0.35253 -11.888   <2e-16 ***
AGC          0.47310    0.03159  14.974   <2e-16 ***
Scan2       -0.73568    0.34658  -2.123   0.0338 *
Slew2        0.83928    0.36783   2.282   0.0225 *
Slew3       -1.03717    0.41592  -2.494   0.0126 *
Slew4       -0.29220    0.38594  -0.757   0.4490
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 1889.82  on 1611  degrees of freedom
Residual deviance:  600.54  on 1606  degrees of freedom
AIC: 612.54

Number of Fisher Scoring iterations: 7
```

Figure 3a Regression Coefficients for S-band

The regression coefficients for our reduced model for the conic-scan antenna ACU autotracking on C-band for S- and C-band carriers are shown in figure 3a and b respectively. The full model for received C-band had significant interaction terms of the *Scan* level 2 and *Slew* level 2. You can see that these are statistically significant. Slew level 4 was excluded due to singularities and the model results show that *Slew* level 3 is not statistically significant. We could exclude this *Slew* factor level from the model, but will not as it would eliminate samples of angles and gain. We keep the models as is. The full S-band carrier model shows Slew levels 3 and 4 are statically insignificant. Note that each model has very different predictor coefficients indicating the parameters' influences in tracking are distinct to receiver band. These means of autotrack counts are the first indication of an inherent difference in tracking mode in distinct bands.

```
#C-band:
> summary(Clogit)

Call:
glm(formula = AT ~ AGC + Scan * Slew, family = binomial, data = Cband1)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.7604  -0.4825  -0.3694  -0.1146   3.6362

Coefficients: (1 not defined because of singularities)
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -2.887123   0.111632 -25.863  < 2e-16 ***
AGC          0.135242   0.006999  19.323  < 2e-16 ***
Scan2       -2.381988   0.342673  -6.951 3.62e-12 ***
Slew2       -4.683895   1.036357  -4.520 6.20e-06 ***
Slew3       -0.048593   0.196259  -0.248    0.804
Scan2:Slew2  5.808177   1.130320   5.139 2.77e-07 ***
Scan2:Slew3        NA         NA      NA       NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 2939.2  on 3408  degrees of freedom
Residual deviance: 2075.8  on 3403  degrees of freedom
AIC: 2087.8

Number of Fisher Scoring iterations: 8
```

Figure 3b Regression Coefficients for C-band

```
> summary(Clogit)

Call:
glm(formula = AT ~ CAGC + Scan * Slew, family = binomial, data = Cband1)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-2.2090  -0.4525  -0.4367  -0.1358   3.7246

Coefficients: (1 not defined because of singularities)
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -2.22731    0.09550 -23.322  < 2e-16 ***
CAGC2        1.45482    0.12310  11.818  < 2e-16 ***
CAGC3        2.20250    0.18824  11.701  < 2e-16 ***
CAGC4        4.57597    0.39782  11.503  < 2e-16 ***
CAGC5        4.92708    0.60193   8.185 2.71e-16 ***
Scan2       -2.37934    0.34040  -6.990 2.75e-12 ***
Slew2       -4.70823    1.05230  -4.474 7.67e-06 ***
Slew3       -0.07469    0.19692  -0.379    0.704
Scan2:Slew2  5.95483    1.14581   5.197 2.02e-07 ***
Scan2:Slew3       NA         NA      NA       NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 2939.2  on 3408  degrees of freedom
Residual deviance: 2118.2  on 3400  degrees of freedom
AIC: 2136.2

Number of Fisher Scoring iterations: 8
```

Figure 4 Alternate Model Coefficients for C-band

We could build a logistics model in which we eliminate the continuous parameters and instead categorize the AGC, as higher power levels should increase probability of autotrack. We examined such models and example results are shown in figure 4 for C-band carrier. It is clear that probability of autotrack mode is more sensitive to higher signal levels. But this

categorization of AGC does not improve the model's explanatory power, as is evidenced by the AIC score (a measure of the relative quality of the statistical model) compared to the AIC for the continuous AGC in figure 3b.; but does show sensitivity to autotrack increases with gain, which is useful information. For plotting purposes of estimating the probability density and cumulative density functions we employ continuous parameters. Figure 5a and b show these plots for S- and C-band carriers, respectively.

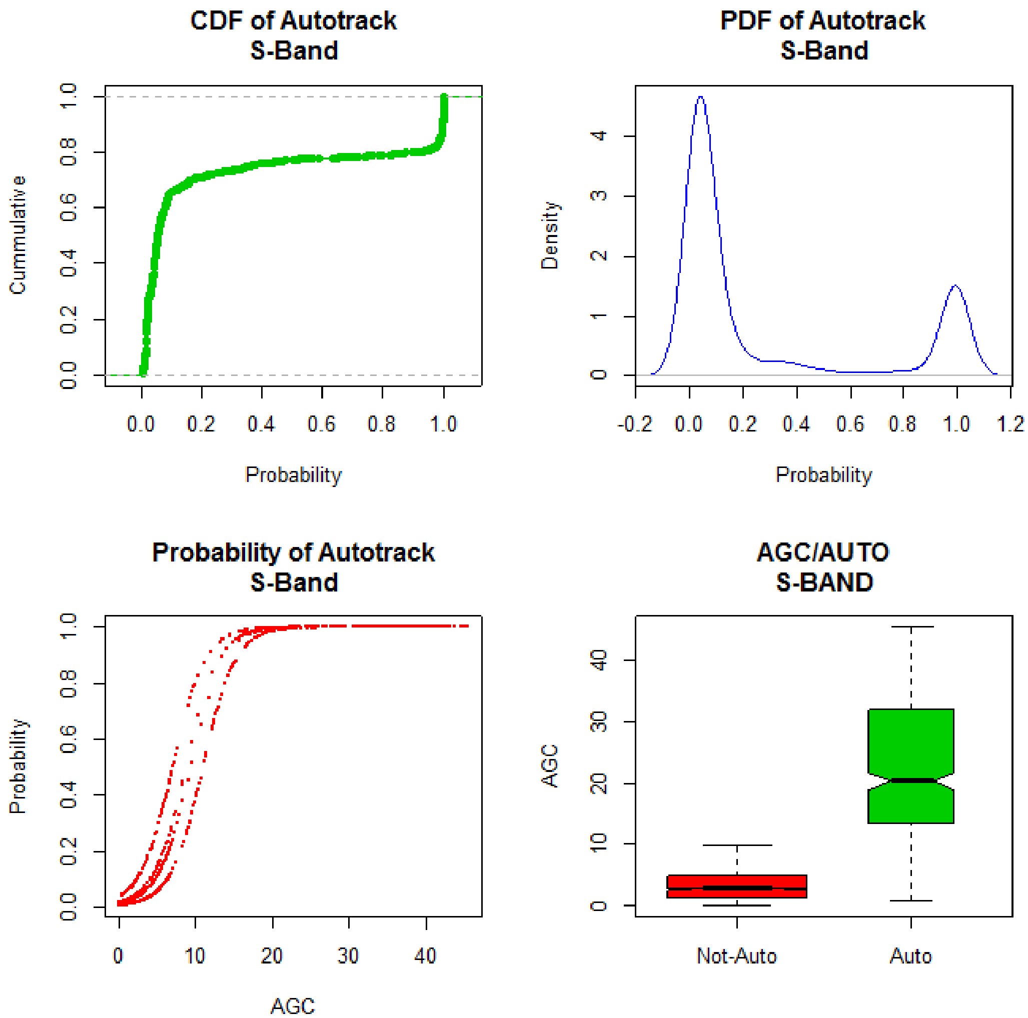


Figure 5a S-band Statistical Plots

Note the probability density (PDF) for both bands show negative probabilities; this is due to 'kernel smoothing' of the histogram. We can eliminate this by employing a histogram which shows the same information. I prefer to leave it as is, and realize the high density near probability zero is 'smoothed' into negative probabilities. Note too that receiving the S-band carrier results in a clear multimode PDF, with concentrations around p = 0 and p = 1, as expected given the high slope of the 'S- curve' (probability of autotrack). The C-band PDF has a less steep S-curve, with a greater dispersion. The box-plot shows a slight overlap in auto-track vs. not-auto-track near 10dB AGC.

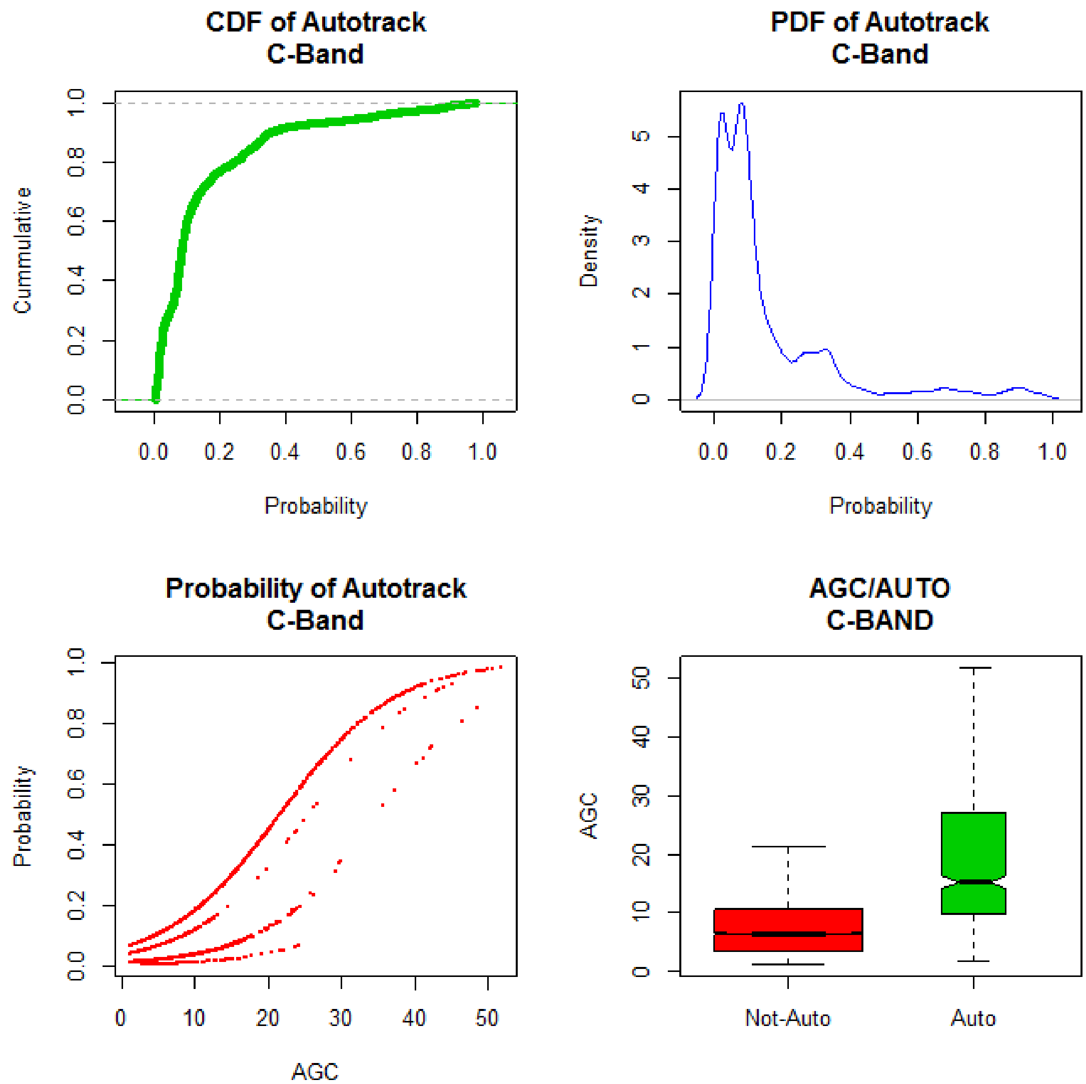


Figure 5b C-band Statistical Plots

Note the appearance in Figures 5a and 5b of what appears to be four probability curves for both bands. For S-band carriers there are actually five; one is hidden by overlap. With four slew and two scan rates we'd expect eight curves, but not all combinations were used. These separate curves are partitioned over combinations *Slew* and *Scan* for each AGC. A plot of a probability and logit vs. AGC at slew levels 1 and 2 at scan 1 is shown in figure 5c. The logit plot being linear over the AGC is a verification that the log-odds is linear w.r.t. the AGC predictor partitioned over the control set predictors. This linearity of the logit vs. predictor is an assumption essential to our model. Not only can we infer a difference in the ACU's ability to track over the two bands, but we can also infer that the ACU tracked C-band carrier less often in autotrack mode than it did S-band carrier. The number of auto-track states, as well as the gentle slope of the probability curve and the overlap of 'Not-Auto' 'Auto' around 10dB indicate a poor tracking profile in C-band of C-band carrier.

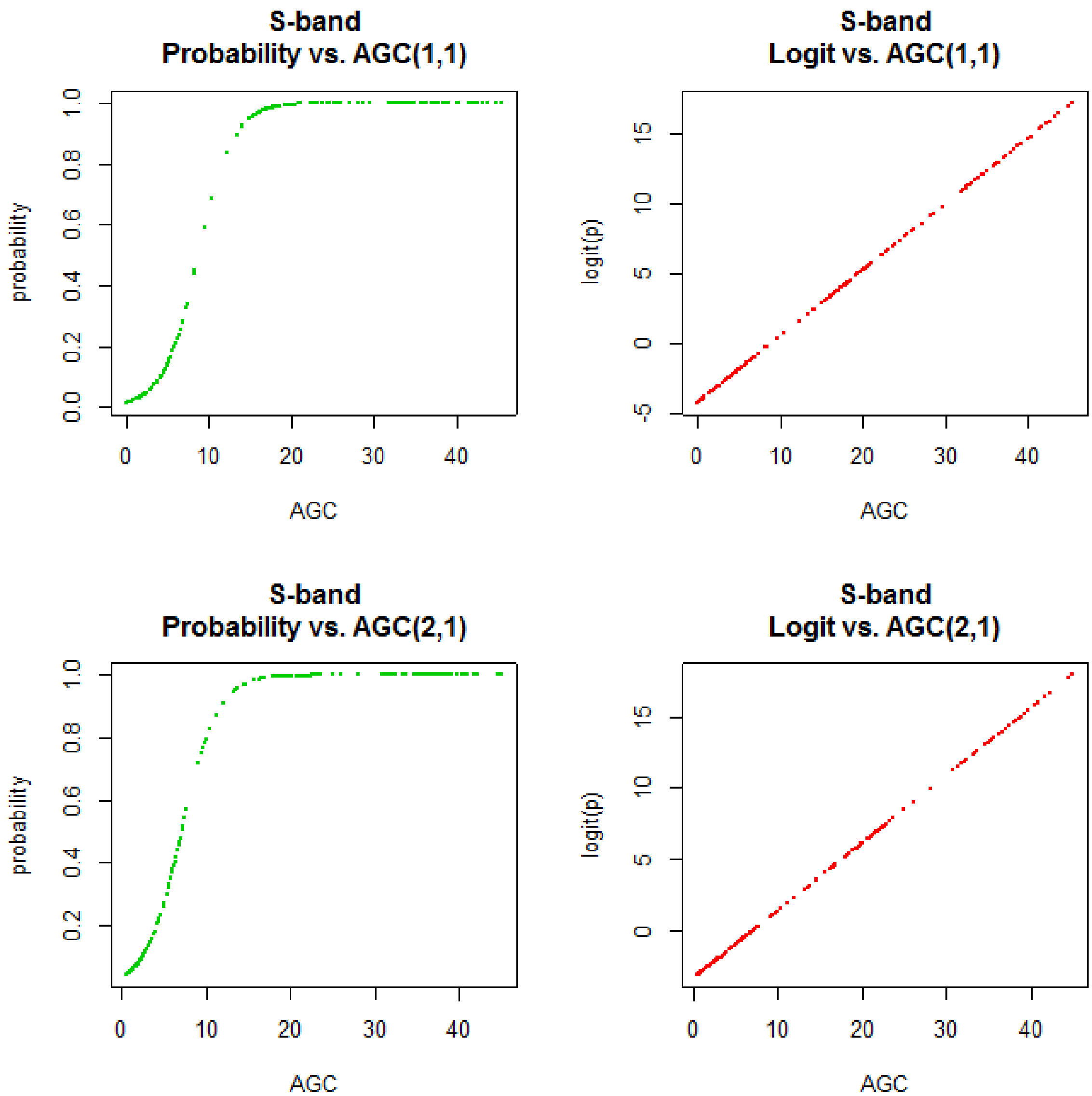


Figure 5c Probability and Logit Curves for S-band

SUMMARY AND CONCLUSION

This paper presents a rather standard synthesis of a tracking filter using observed data and antenna tracking controls to model an autotracking predictor model that explores statistical measures on observations to decide the probability of autotracking, based on received signal strength and the state of control of the ACU. That there is an error in tracking is evidenced by the data, statistics and models developed in this and the preceding paper mentioned above. Our analysis and models confirm out conclusions reached in the companion paper that we reject the null, H_0 and accept the alternative hypothesis:

H_1: *AUT autotracks statistically differently on C-band while receiving S- or C-band carriers.*

There is also a practical difference. The probability of autotrack was far more decisive while receiving an S-band carrier. As to the cause, we make no hypothesis. The purpose was to determine if there was a significant statistical difference in the probability of autotracking in C-band when receiving C- and S-band carriers. The antenna modes are not close to identically probable within or across bands. This is not a general conclusion, but rather particular to the test configuration. We could extend our synthesis: this model, along with a tracking-error model, together form essential systems of a stochastic tracking filter. Such work must await another opportunity.

ACKNOWLEDGEMENTS

The author would like to thank Kip Temple, 412th Test Wing Telemetry Technical Expert for allowing his assistance to data processing which precipitated these analyses and modeling, and for his commenting on content and presentation. The author also thanks Tim Chalfant, 412th Test Wing Director of Instrumentation Engineering for the time to develop these models and write this paper and Abigail L. Reuter Deputy Director, 412th Electronic Warfare Group for her critique and suggestions.

REFERENCES AND RELATED PAPERS:

1. Geometric Model for Tracker-Target Look Angles and Line-of-Sight Distance, Daniel T. Laird, ITC Las Vegas NV, 19-22 October 2015
2. Analysis of Covariance on Antenna Control Unit Tracking with Linear Error Regression Model, Daniel T. Laird, ITC Las Vegas NV, 19-22 October 2015.
3. Telemetry Tracking Antennas at AFTC, How It's Done, Kip Temple, Engineering Training Brief, AFTC Edwards AFB CA, 2014.
4. Logistic Regression Models, Joseph M. Hilbe, CRC Press; Taylor & Francis Group, Boca Raton, FL, 2009.
5. The R Book, Michael J. Crawley; Wiley & Sons, United Kingdom, 2007.
6. Stochastic Processes and Filter Theory, Andrew H. Jazwinski, Academic Press; New York & United Kingdom, 1970.
7. Multivariate Statistical Inference, Narayan C. Giri, Academic Press; New York & United Kingdom, 1977.
8. Functional Analysis, Walter Rudin, McGraw Hill 2nd Edition; New York & London, 1991.